Rateless Distributed Source Code Design

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MobiMedia 2009

Outline

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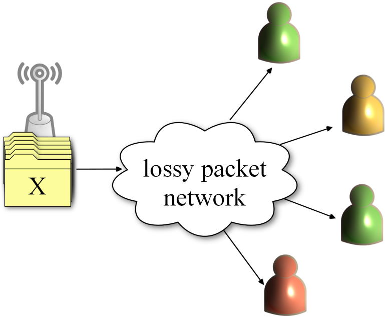
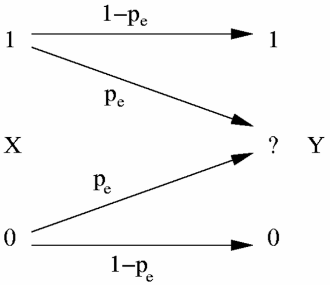
1. Fountain codes: state of the art
2. Rateless coding with side info
3. Fountain coding with multiple source nodes

Multicast transmission in a lossy packet network



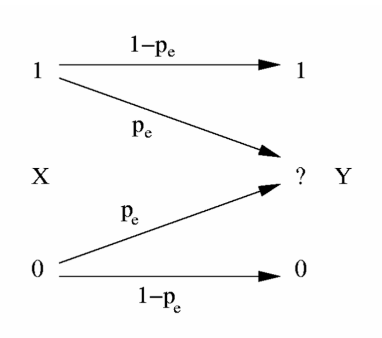
- Receivers experience different and dynamically changing packet loss rates.
- Wireless erasure networks / mobile environments.
- ARQ/Feedback implosion.

Erasure coding



- Erasure codes (MDS - Reed-Solomon)?
  - low operational complexity (mobile devices: computational resources and battery power)
    - Sparse graph codes coupled with **belief propagation** (BP) algorithm: LDPC, Turbo, LDGM, IRA...

Erasure coding



- Erasure codes (MDS - Reed-Solomon)?
  - support for a wide range of (and dynamically changing) packet loss rates
    - code rate= ??

Digital fountain

$\#$ encoding packets$=\infty$, code rate $= 0$

Practical fountain codes

Fountain codes are:

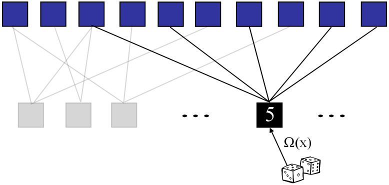
- rateless - a potentially limitless amount of encoding packets.
- computationally efficient and scalable - fast and parallelizable algorithms.
- nearly optimal - reliable data reconstruction from any set of encoding packets only slightly greater than the size of the original message.

Standardization

Digital Fountain's Raptor FEC has been adopted by:

- 3GPP Multimedia Broadcast/Multicast
- DVB-h IP datacast to handheld devices
- IETF Reliable Multicast Transport (RMT)

LT codes



- (Luby 2002), $LT(k,\Omega(x))$ code ensemble: k- size of the message, $\Omega(x) = \sum_{d=1}^{k} \Omega_d x^d$ probability distribution on $\{1,2,\ldots,k\}$ (gen. poly.)
  - Sample an output degree d with probability Ω_d.
  - Sample d distinct data packets uniformly at random and XOR them.

LT codes achieve capacity

Fact

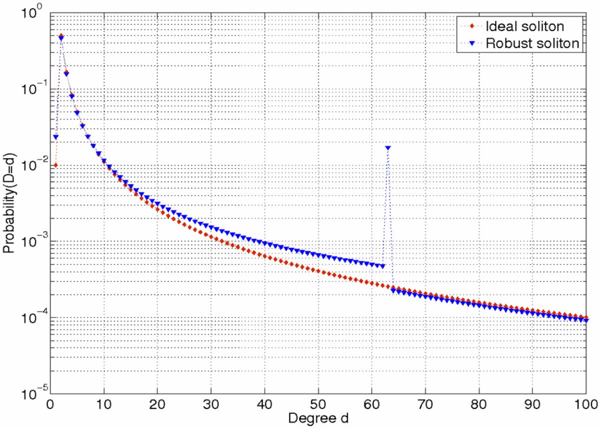
There exist sequences of LT code ensembles $LT(k, \Omega^{(k)}(x))$ which achieve capacity regardless of the erasure probability of the channel (universality) with computational cost of $\mathcal{O}(k \log k)$.

- $\Omega^{(k)}(x)$ converges pointwise to limiting soliton distribution:

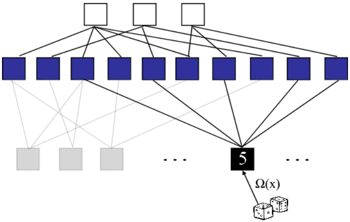
$$\Psi_\infty(x) = \sum_{d=2}^{\infty} \frac{x^d}{d(d-1)}$$

- Small perturbations suffice at finite lengths: robust soliton.

Soliton distributions

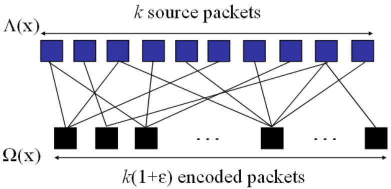


Raptor codes



- (Shokrollahi 2006), $\Omega(x)$ capped at a max. degree d_{max} as $k \to \infty$ (lowers computational cost to $\mathcal{O}(k)$ but introduces an error floor) - decode fraction $1-\delta$.
- Error floor is removed by an outer very high rate LDPC code - sufficient redundancy to finish off decoding.

Decoding graph



ε: code overhead

BP decoding asymptotic analysis

- (Luby, Mitzenmacher, Shokrollahi 1998) AND-OR tree evaluation
- Generalized todensity evolution techniques (Richardson, Urbanke, *MCT*, 2008)
- Recipe for fountain code design:
  - formulate a particular version of density evolution - set of recursive equations
  - generate an optimization procedure based on the density evolution equations (typically LP).

Optimisation of $\Omega(x)$

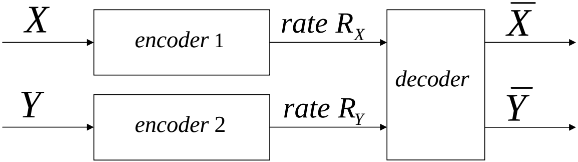
Fix $d_{\max}$ and δ and minimise ε:

$$\begin{array}{rcl} \mathrm{LP}: & & \min \sum_{d}^{d_{max}} \frac{\omega_d}{d} (\sim 1+\varepsilon) \\ \sum_{d=1}^{d_{max}} \omega_d (1-y_i)^{d-1} & \geq & -\ln y_i,\ i \in \{1,2,\ldots,m\}, \\ \omega_d & \geq & 0,\ d \in \{1,2,\ldots,d_{\max}\}. \end{array}$$

- $1 = y_1 > y_2 > \cdots > y_m = \delta$ are m equidistant points on $[\delta, 1]$, δ is the desired error rate, and d_{max} is the max. degree.

Outline

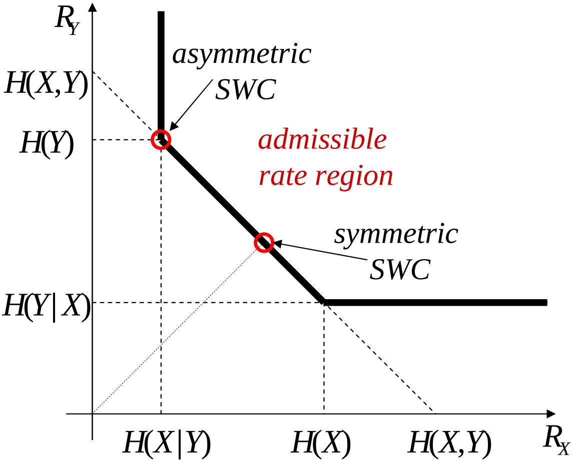
Slepian-Wolf Coding (SWC)



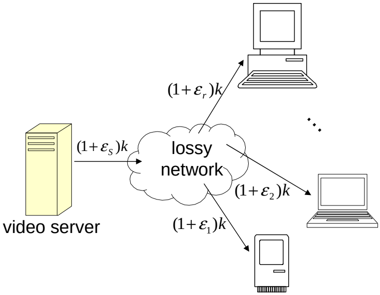
The admissible rate region for the pairs of rates (R_X, R_Y) is given by:

$$\begin{aligned} R_X &\geq H(X|Y) \\ R_Y &\geq H(Y|X) \\ R_X + R_Y &\geq H(X,Y). \end{aligned}$$

Slepian-Wolf Coding (SWC)



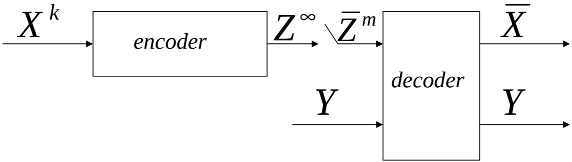
Scalable video multicast



- scalable video over loss - prone wireless networks
- Single channel code for both:
  - video compression (Slepian-Wolf coding)
  - packet loss protection

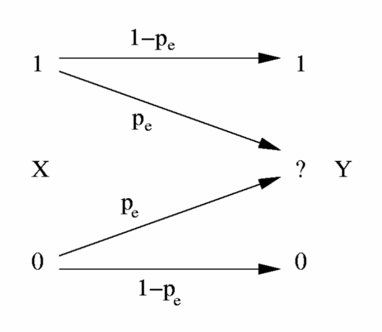
Xu, Stankovic, Xiong - *IEEE JSAC May 2007*.

Rateless Asymmetric SWC



$$m \geq kH(X \mid Y)$$

"Erasure correlation" SWC

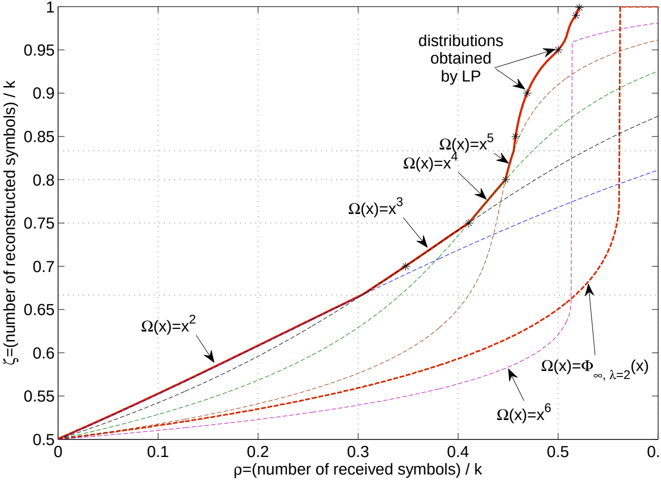


- Y is the output of an erasure channel when X is the input.
- Receivers have a priori knowledge of a number of data packets (transmission from other sources ?).

Asymmetric SWC - side information

- Systematic Raptor - *Fresia, Vandendorpe (Globecom 2007).*
- Non-systematic LT (shifted robust soliton) - *Agarwal, Hagedorn, Trachtenberg (ITA Workshop 2008).*
- Both systematic and non-systematic: asymptotic analysis and design - *Sejdinovic, Piechocki, Doufexi, Ismail (IEEE ICC 2008, IEEE Trans. Wireless Commun. 2009).*
- IR-HARQ with LDPC/Fountain codes - *Sejdinovic, Ponnampalam, Piechocki, Doufexi (IEEE WCNC 2008).*

Asymptotic code design with side info



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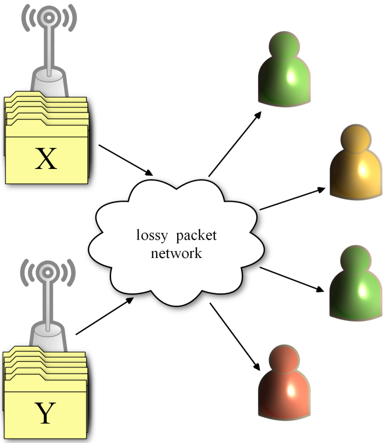
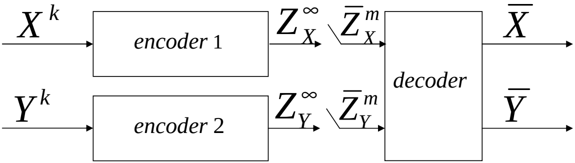
Multiple source nodes



Rateless Symmetric SWC



$$m \geq \frac{kH(X,Y)}{2}$$

Multiple source nodes

- Obstacles:
  - No cooperation or centralized controller
  - Each source node produces localized encoding packets.
- Questions:
  - How to perform (small) decentralized encoding taskssuch that the resulting decoding problemis well-behaved?
  - Can relay help by combining data from multiple sources?

DE: General case

- Packets dispersed across s source nodes.
- Sets of packets available at different nodes are not necessarily disjoint nor of equal size.
- Each source node oblivious of which packets are available at other source nodes.
- *IEEE Commun. Letters 2009* (under review, with Piechocki, Doufexi, Ismail)

DE: General case

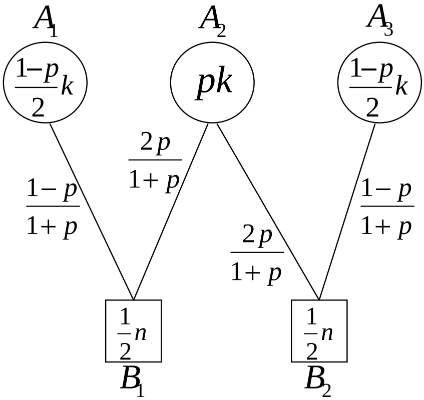
- Rigorous asymptotic analysis for many data dissemination scenarios.
- Generalized DE leads to a simple asymptotic code design yielding both multiterminal source coding and channel coding gains.
- Easy modification to include the case of informed collector node, i.e., decoder side information.
- Amenable to extension for noisy channels and general belief propagation algorithm (channels like BSC and BIAWGNC).

Symmetric SWC with LT codes

Example

Source nodes S_1 and S_2 are trying to multicast k packets. Each source node contains $t > k/2$ packets, but is oblivious of which t packets are available at other node. Source nodes use $LT(t, \Omega(x))$ ensemble and receiver obtains $n/2$ encoding packets from each source node.

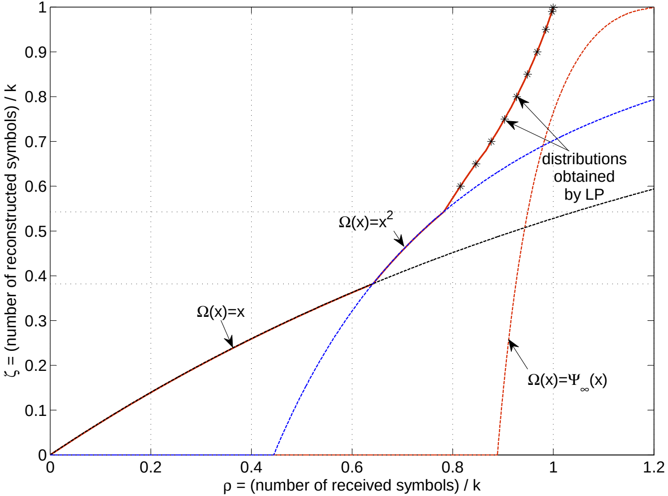
Symmetric SWC - DDLT



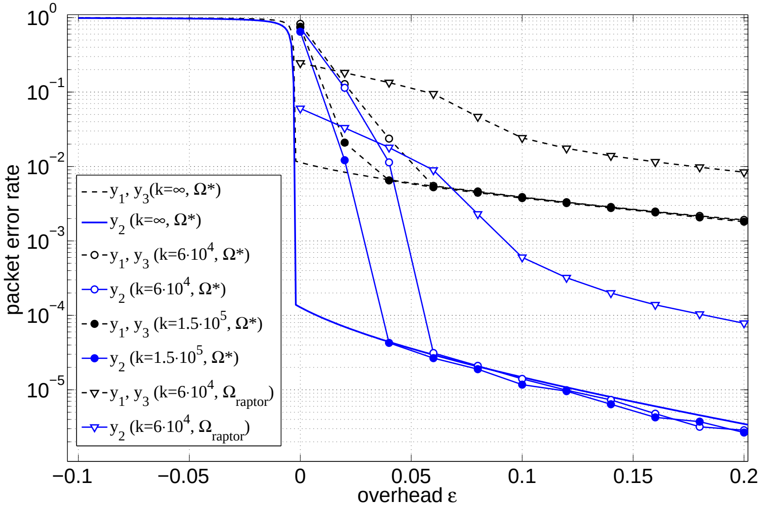
Optimization of $\Omega(x)$, $p = 1/3$

$$
\begin{aligned}
\mathrm{LP}: \quad & \min \sum_{d=1}^{d_{\max}} \frac{\omega_d}{d} \\
\frac{1}{1+p}\omega\left(1 - \frac{1-p}{1+p}y_i - \frac{2p}{1+p}y_i^2\right) &\geq -\ln y_i, \;\; i \in \{1, 2, \ldots, m\}, \\
\omega_d &\geq 0, \; d \in \{1, 2, \ldots, d_{\max}\}.
\end{aligned}
$$

Intermediate performance



Numerical results



Limiting distribution

- Perturbation of the Limiting soliton for correlated data.

$$\Omega(x) = -\frac{2(1+p)}{1+3p}\int_0^x \ln\frac{\sqrt{t(p^2-1)+1}-p}{1-p}dt,\ x\in[0,1). \qquad (1)$$

Fact

When two terminals contain erasure correlated data, fountain coding can still achieve information theoretic limits, provided that the code design is appropriately modified.

Summary

- Overview of fountain coding - LT, Raptor codes
- DSC with fountain codes
- Generic setting with multiple source nodes
- Symmetric SWC - perturbation of soliton distribution achieves SW limit.